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The role of temperature in the Bloch oscillator problem

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Abstract
The static and high-frequency differential conductivity of a one-dimensional superlattice with parabolic miniband, in which the dispersion law is assumed to be parabolic up to the Brillouin zone edge, is investigated theoretically. Unlike the earlier published works, devoted to this problem, the novel formula for the static current density contains temperature dependence, which leads to the current maximum shift to the low field side with increasing temperature. The high-frequency differential conductivity response properties including the temperature dependence are examined and opportunities of creating a terahertz oscillator on Bloch electron oscillations in such superlattices are discussed. Analysis shows that superlattices with parabolic miniband dispersion law may be used for the generation and amplification of terahertz fields only at very low temperatures ($T \to 0$).

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1. Introduction

An idea of a THz Bloch oscillator based on semiconductor superlattices (SLs) is discussed intensely (see [1–5] and references therein). The idea is based on the following arguments. If a high enough dc electric field E is applied along the SL axis, then an electron traveling from one Brillouin zone edge to another (within one energy band) almost without scattering executes Bloch oscillations (BO) with frequency $\Omega = eEd/\hbar$ (d is the SL spatial period). At $E = 3$ kV cm^{-1} and $d = 2 \times 10^{-7}$ cm we have $\Omega = 1$ THz, and the frequency may be tuned continuously by an applied electric field. However, the existence of the BO is not sufficient to generate radiation. The negative differential conductivity (NDC) near the Bloch frequency is needed. Studying NDC is a traditional problem in investigations on transport in SL (see [6], for example), which were began in the pioneer work by Esaki and Tsu [7]. One of the conditions of THz field generation and amplification at the Bloch frequency harmonics is the existence of high-frequency NDC in the current–voltage curve with positive static differential conductivity. It was shown [4, 8] that this condition can be realized in the case of extremely low temperatures, in particular, for a SL with 'parabolic miniband'. We mean a dispersion law in the form of a truncated parabola (i.e., the dispersion law is assumed to be parabolic up to the Brillouin zone edge). Note that already in [7, 9] a dispersion law constructed from direct and overturned parabolas was considered (together with the cosine dispersion law). The parabolic dispersion law in question is a special case of that model. Such models were studied [7] in reference to specific superlattices by discussing the preparation methods. Our aim is to find the temperature dependence of NDC, to analyze the temperature effect on the 'mean' electron BO and to discuss in what extent the idea mentioned [4, 8] is realistic at finite temperatures.

2. Static distribution function and current–voltage characteristic

The electron energy in the lowest parabolic miniband of the SL is

$$\mathcal{E}(\mathbf{p}) = \frac{\mathbf{p}_\perp}{2m_\perp} + \varepsilon(p), \tag{1}$$

where $\mathbf{p}_\perp$ is quasimomentum, $m_\perp$ is electron effective mass in the SL layer plane,

$$\varepsilon(p) = \frac{\Delta d^2}{\pi^2\hbar^2}\cdot\frac{p^2}{2}, \qquad -\frac{\pi\hbar}{d} \leqslant p \leqslant \frac{\pi\hbar}{d}, \tag{2}$$

p is electron quasimomentum along the SL axis and Δ is the doubled miniband width. The longitudinal energy $\varepsilon(p)$ may be written as the Fourier series (as a set of partial cosine minibands)

$$\varepsilon(p) = \frac{1}{2}\sum_{k=1}^{\infty}\Delta_k\left(1 - \cos\left(\frac{kpd}{\hbar}\right)\right), \tag{3}$$

where $\Delta_k = (-1)^{k+1}(4\Delta)/(\pi^4k^2)$ is the width of the partial cosine miniband.

In the quasi-classical situation ($\Delta \gg eEd, \hbar/\tau$, where τ is the electron momentum relaxation time and e is the electron charge), the current density in electric field $\mathbf{E}^{\text{tot}}(t)$ may be found by solving the Boltzmann equation with the collision integral within the τ-approximation:

$$\frac{\partial F(\mathbf{p},t)}{\partial t} + \left(e\mathbf{E}^{\text{tot}}(t), \frac{\partial F(\mathbf{p},t)}{\partial \mathbf{p}}\right) = \frac{F_0(\mathbf{p}) - F(\mathbf{p},t)}{\tau}, \tag{4}$$

where $F_0(\mathbf{p})$ is the equilibrium electron distribution function and $F(\mathbf{p},t) \equiv F(\mathbf{p}_\perp, p, t)$ is the unknown distribution function perturbed due to the electric field. Below we use dimensionless variables by changing $p\cdot d/(\pi\cdot\hbar) \to p$, $\mathbf{E}^{\text{tot}}ed\tau/(\pi\cdot\hbar) \to \mathbf{E}^{\text{tot}}$, $T/\Delta \to T$, $t/\tau \to t$ (T is the temperature in energy units).

Below we consider the case when the $\mathbf{E}^{\text{tot}}(t)$ field is directed along the SL lattice. As shown [9], the dependence of $F(\mathbf{p},t)$ on $\mathbf{p}_\perp$ is the same as that of $F_0(\mathbf{p})$. This is because such a model collision integral does not mix the electron degrees of freedom which, therefore, become mutually independent. Equation (4) integrated over $\mathbf{p}_\perp$ takes the form

$$\frac{\partial f(p,t)}{\partial t} + E^{\text{tot}}(t)\frac{\partial f(p,t)}{\partial p} = f_0(p) - f(p,t) \qquad (-1 \leqslant p \leqslant 1). \tag{5}$$

where $f(p,t)$ is the distribution function of longitudinal quasimomenta that satisfies the periodicity condition $f(1,t) = f(-1,t)$. Below that function (as well as $f_0(p)$) is normalized to the carrier density n.

In a static field $E^{\text{tot}}(t) = E = \text{const}$, and denoting $f(p) = f_c(p, E, T)$, we get

$$E\frac{\mathrm{d}f_c}{\mathrm{d}p} = f_0 - f_c \qquad (-1 \leqslant p \leqslant 1). \tag{6}$$

We consider the non-degenerate electron gas, so that

$$f_0(p, T) = 2n\left[\sqrt{2\pi T}\,\mathrm{erf}\left(\frac{1}{\sqrt{2T}}\right)\right]^{-1}\exp\left(-\frac{p^2}{2T}\right) \tag{7}$$

where $\mathrm{erf}(z)$ is the error function. In the low temperature limit ($T \to 0$) the relation (7) reduces to the function used in [8]: $g_0(p) = 2n\delta(p)$.

The exact solution of equation (6) with periodicity condition, $f_c(-1) = f_c(1)$, takes the form [10]

$$\begin{aligned} f_c(p, E, T) = {}& \frac{n}{E\,\mathrm{erf}(1/\sqrt{2T})}\exp\left(\frac{T}{2E^2} - \frac{p}{E}\right)\left\{\mathrm{erf}\left(\frac{p}{\sqrt{2T}} - \frac{\sqrt{T}}{\sqrt{2}E}\right)\right. \\ & - \left[\exp\left(\frac{2}{E}\right) - 1\right]^{-1}\mathrm{erf}\left(\frac{\sqrt{T}}{\sqrt{2}E} - \frac{1}{\sqrt{2T}}\right) \\ & \left. + \left[1 - \exp\left(-\frac{2}{E}\right)\right]^{-1}\mathrm{erf}\left(\frac{\sqrt{T}}{\sqrt{2}E} + \frac{1}{\sqrt{2T}}\right)\right\} \qquad (-1 \leqslant p \leqslant 1). \end{aligned} \tag{8}$$

In the limiting case $E \to 0$ (8) reduces to (7). In another limiting case, $T \to 0$, we get the distribution function found in [8]:

$$g(p, E) = \frac{2n}{E}\exp\left(-\frac{p}{E}\right)\begin{cases}[1 - \exp(-2/E)]^{-1}, & 0 < p \leqslant 1, \\ [\exp(2/E) - 1]^{-1}, & -1 \leqslant p < 0.\end{cases} \tag{9}$$

The function (8) satisfies the same normalization condition as the equilibrium function f_0

$$\frac{1}{2}\int_{-1}^{1} f_c(p, E, T)\mathrm{d}p = n \tag{10}$$

and, therefore, it makes the integral on right-hand side of formula (6) vanish. Besides, the integral on left-hand side of the Boltzmann equation (6) vanishes too, because of the periodicity condition mentioned. The distribution function $f_c(p, E, T)$ at several values of E and T is shown in figure 1.

The current density j in the direction of the SL axis can be found (in dimensional units) by a conventional way

$$j = \frac{ed}{2\pi\hbar m}\int_{-\pi\hbar/d}^{\pi\hbar/d} p f_c(p)\,\mathrm{d}p. \tag{11}$$

By substituting function (8) into (11) we get

$$\begin{aligned} j(E, T) = E + {}& \left[2\mathrm{erf}\left(\frac{1}{\sqrt{2T}}\right)\sinh\left(\frac{1}{E}\right)\right]^{-1}\exp\left(\frac{T}{2E^2}\right) \\ & \times\left[\mathrm{erf}\left(\frac{\sqrt{T}}{E\sqrt{2}} - \frac{1}{\sqrt{2T}}\right) - \mathrm{erf}\left(\frac{\sqrt{T}}{E\sqrt{2}} + \frac{1}{\sqrt{2T}}\right)\right]. \end{aligned} \tag{12}$$

Here j is expressed in units of $j_0 = ne\Delta d/\pi\hbar$, while all the quantities are written in dimensionless form.

Equation (12) determines the current–voltage characteristic for the parabolic miniband SL by taking into account the current-density temperature dependence.

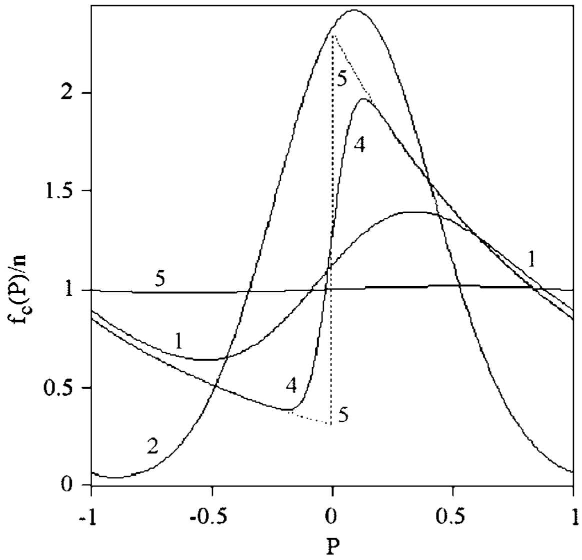

Figure 1. Distribution function $f_c(p)$ at various values of the driving field and temperature: (1) $E = 1, T = 0.1$; (2) $E = 0.1, T = 0.1$; (3) $E = 2, T = 2$; (4) $E = 1, T = 0.005$. The dashed curve 5 represents function $g(p)$ at $E = 1$.

To warrant numerical stability we present formula (12) in the following form:

$$j(E,T) = E\sigma(E,T), \qquad \sigma(E,T) = 1 - \sqrt{\frac{2}{\pi T}}\frac{\exp(-0.5/T) + A(E,T)}{\operatorname{erf}(1/\sqrt{2T})}, \tag{13}$$

where $\sigma(E,T)$ is the conductivity and

$$A(E,T) = \frac{E^2}{T\sinh(1/E)}\int_0^{1/E}\exp\left(-\frac{s^2E^2}{2T}\right)s\sinh s\,\mathrm{d}s. \tag{14}$$

The value of $A(E,T)$ can be estimated numerically with high accuracy.

Expanding the exponent in a power series we get

$$A(E,T) = \frac{1}{T}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!!}\frac{G_n(E)}{T^n}, \tag{15}$$

where functions $G_n(E)$ are defined by the recurrent formula

$$G_0 = E\coth\left(\frac{1}{E}\right) - E^2, \qquad G_n = G_0 + 2nE^2[(2n+1)G_{n-1} - 1]. \tag{16}$$

As $G_n(E) \in [0, 1/(2n+3))$, series (15) converges quickly. As numerical experiments show, the first four terms of series (15) give a good approximation at $T > 0.5$.

At $|E| \to 0$ we have $A(E,T) \to 0$, so in low fields ($|E| \ll 1$) in the linear approximation on E we have

$$j(E,T) = E\left(1 - \sqrt{\frac{2}{\pi T}}\frac{\exp(-0.5/T)}{\operatorname{erf}(1/\sqrt{2T})}\right) = E\left\langle\frac{p^2}{T}\right\rangle_0, \tag{17}$$

where angle brackets mean averaging over the equilibrium distribution. Note that the conductivity temperature dependence in low fields (the expression within round brackets in (17)) is close to the analogous dependence for the miniband cosine model ($I_1(1/2T)/I_0(1/2T)$, $I_n(z)$ being the modified Bessel function).

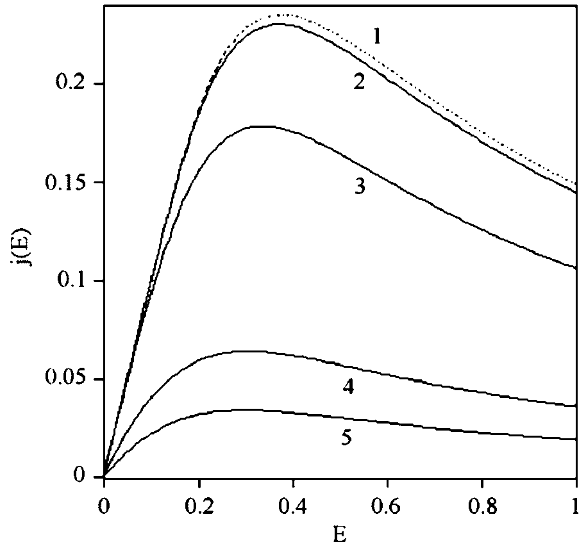

Figure 2. Current–voltage characteristic at different values of temperature: (1) $T = 0$; (2) $T = 0.01$; (3) $T = 0.1$; (4) $T = 0.5$; (5) $T = 1$.

In high fields ($|E| > 1$) we have

$$\sigma(E,T) \approx \frac{\sqrt{2/\pi}}{E^2 T\sqrt{T}\,\mathrm{erf}(1/\sqrt{2T})} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n)!!}\frac{D_n}{T^n}, \qquad D_n = \sum_{k=2}^{n+2} \frac{2^{2k}(2n+1)!B_{2k}}{(2k)!(2n-2k+5)!}, \tag{18}$$

where B_m are Bernoulli numbers.

For low temperatures ($T \ll 1$), using (14), we get

$$\sigma(E,T) \approx 1 - \frac{1}{\mathrm{erf}(1/\sqrt{2T})}\left[\frac{1}{E\sinh(1/E)}\exp\left(\frac{T}{2E^2}\right) + \sqrt{\frac{2}{\pi T}}\exp\left(-\frac{1}{2T}\right)\right]. \tag{19}$$

As numerical experiments show, formula (19) gives good approximation at $T < 0.07$. In limiting case $T \to 0$ from (19) we get the expression that was found in [8].

$$j = j(E) = E - \frac{1}{\sinh(1/E)}. \tag{20}$$

From equations (17), (18) it follows that $j \sim E$ at $|E| \ll 1$ and $j \sim 1/E$ at $|E| \gg 1$. Therefore, at fixed temperature $T = fix$ the function $j(E,T)$ reaches its maximum at some value $E = E_C(T) > 0$ and the negative differential conductivity is realized at $E > E_C(T)$ (see figure 2).

Note that $E_C(T)$ decreases with increasing temperature. Essentially, that E_C value does not depend on the temperature at all in the cosine model: $E_C = 1/\pi \approx 0.318$.

The parametric representation of dependence $E_C(T)$ is defined by equation $\sigma_d = 0$, where $\sigma_d = \partial j/\partial E$ is the differential conductivity. Using equations (13) and (14), we get

$$\sigma_d(E,T) = 1 + \frac{1}{E^2}\left\{\left[E\coth\left(\frac{1}{E}\right) - T\right][\sigma(E,T) - 1] - \sqrt{\frac{2T}{\pi}}\frac{1}{\mathrm{erf}(1/\sqrt{2T})}\exp\left(-\frac{1}{2T}\right)\right\}, \tag{21}$$

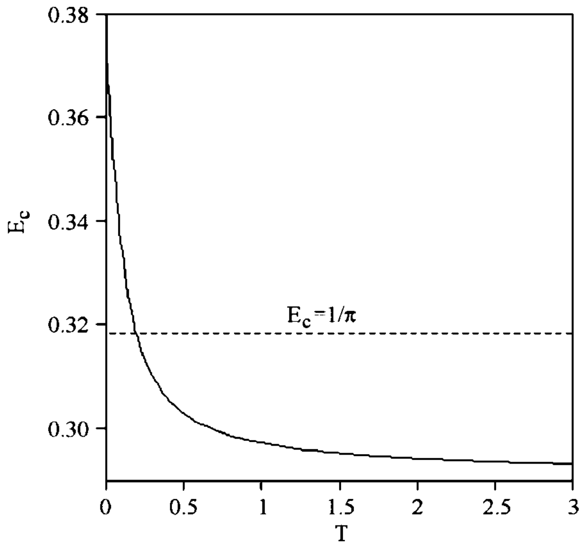

Figure 3. The dependence $E = E_C(T)$. The dashed curve $E_C = 1/\pi$ represents E_C for the cosine model.

Thus function $E_C(T)$ is defined by implicitly by the equation

$$E^2\sqrt{\frac{\pi T}{2}}\operatorname{erf}\left(\frac{1}{\sqrt{2T}}\right) + TA(E,T) = E\coth\left(\frac{1}{E}\right)\left[A(E,T) + \exp\left(-\frac{1}{2T}\right)\right] \tag{22}$$

and it is sufficient to solve this equation at $E > 0$.

The numerical solution of equation (22) at E versus T is presented in figure 3.

Note that the dependence $E = E_C(T)$ is monotone, so the inverse function $T_C = T_C(E)$ exists. To investigate the behavior of the function $T_C(E)$, first consider the case of high temperatures $T \gg 1$. Expanding all functions in a power series on $1/T$ and neglecting all terms $o(1/T^2)$, we get

$$T_C \approx \frac{(45E^4 + 22.5E^2 + 1.8)\tanh^2(1/E) - (36E + 3E)\tanh(1/E) - 9E^2 - 1.5}{(9E^2 + 4)\tanh^2(1/E) - 6E\tanh(1/E) - 3} \tag{23}$$

By that

$$\lim_{E\to E_1+0} T_C(E) = +\infty, \tag{24}$$

where $E_1 \approx 0.291\,049\,55$ is the root of equation

$$(9E^2 + 4)\tanh^2\left(\frac{1}{E}\right) - 6E\tanh\left(\frac{1}{E}\right) - 3 = 0. \tag{25}$$

Consider now the case of low temperatures $T \ll 1$. Using (19), we get

$$T_C(E) \approx 2E^2\left[E^2\tanh\left(\frac{1}{E}\right)\sinh\left(\frac{1}{E}\right) - 1\right]\left[1 - 2E\tanh\left(\frac{1}{E}\right)\right]^{-1}. \tag{26}$$

By that

$$\lim_{E\to E_2-0} T_C(E) = 0, \tag{27}$$

where $E_2 \approx 0.373\,681\,745$ is the root of equation

$$E^2 \tanh\left(\frac{1}{E}\right) \sinh\left(\frac{1}{E}\right) = 1. \tag{28}$$

Therefore, the function $E_C(T)$ is defined for $T > 0$ and

$$\lim_{T\to 0} E_C(T) = E_2, \qquad \lim_{T\to +\infty} E_C(T) = E_1. \tag{29}$$

Let us show that the existence of the static NDC is related to the electron Bloch oscillations. The electron quasimomentum in the dc electric field is $p(t) = eEt + p_0(p_0 = p(0))$, while the BO velocity $(\partial\varepsilon/\partial p)$ is

$$V(t) = \frac{2d}{\hbar}\sum_{k=1}^{\infty} k\Delta_k \sin(k\Omega t + kp_0). \tag{30}$$

The $V(t)$ function takes the form of an asymmetric Bloch saw with sloping rise and vertical drop. It is an infinite negative inverse effective mass $(\partial^2\varepsilon/\partial p^2)$ that corresponds to the vertical drops (i.e., Bragg reflections). In the rest of the Brillouin zone the effective mass is positive. According to the Chambers method [11] (see also [7, 9]), the current density can be calculated by the following formula:

$$j = \frac{ed}{2\pi\hbar}\int_{-\pi\hbar/d}^{\pi\hbar/d} \mathrm{d}p_0\, f_0(p_0) \int_0^{\infty} V(t)\,\mathrm{e}^{-\frac{t}{\tau}}\,\frac{\mathrm{d}t}{\tau}. \tag{31}$$

By substituting equations (8) and (30) we get formula (12) for j at $T \neq 0$.

It is obvious that the more BO periods go in between two scattering events the less probable is the phase break and the less is the electron mean drift velocity. At the same time, the BO period decreases with increasing electric field E. Therefore, the electron mean drift velocity and the electron current decrease with increasing the field (at $\Omega\tau > 1$).

Averaging equation (30) over $f_0(p_0)$ gives the BO velocity for a 'mean' electron $\overline{V(t)}$ that depends substantially on temperature. At $T \neq 0$, the $\overline{V(t)}$ remains periodic (with the same period $2\pi/\Omega$ as at $T = 0$), but the features mentioned disappear, while the BO amplitude decreases with rising temperature in complete accordance with the behavior of the current–voltage curve maximum.

Note that the Bloch saw at $T > 0.05$ practically transforms to the sinusoid that corresponds the cosine dispersion law $\left(\varepsilon(p) = \frac{1}{2}\Delta(1 - \cos p)\right)$.

3. High-frequency differential conductivity

In this section, we will determine the induced superlattice current in the presence of an external electric field given by

$$E^{\mathrm{tot}}(t) = E + E_0 \cos\omega t, \tag{32}$$

where ω is measured in unit of τ^{-1}. Within the scope of quasi-classical conditions, the value of E is arbitrary. Assuming the amplitude of variable field E_0 to be much smaller than the static field E, consider the time-dependent field in the linear approximation. The distribution function may be found in a form

$$f(p, E, T, t) = f_c(p, E, T) + f_1(p, E, T, \omega)\exp(-\mathrm{i}\omega t), \tag{33}$$

here $f_1(p, E, \omega)$ satisfies the following equation [12]:

$$E\frac{\partial f_1}{\partial p} + (1 - \mathrm{i}\omega) f_1 = -E_0\frac{\partial f_c}{\partial p} \tag{34}$$

with periodicity condition $f_1(-1, E, \omega) = f_1(1, E, \omega)$ and by

$$\int_{-1}^{1} f_1(p, E, T, \omega)\,\mathrm{d}p = 0. \tag{35}$$

It is easy to show that the required solution is

$$f_1(p, E, T, \omega) = \frac{\mathrm{i}}{\omega}\cdot\frac{E_0}{E}\left[f_c(p, E, T) + f_c\left(p, \frac{E}{1-\mathrm{i}\omega}\right)\right]. \tag{36}$$

With the help of (36) the dynamic (high-frequency) differential conductivity can be found by a conventional way. The result is

$$\sigma_1(E, T, \omega) = \frac{\mathrm{i}}{\omega E}\left[j(E, T) - j\left(\frac{E}{1-\mathrm{i}\omega}, T\right)\right]. \tag{37}$$

From (37) it follows that at $\omega \to 0$ the value $\sigma_1(E, T, \omega)$ tends to static differential conductivity (21)

$$\lim_{\omega\to 0}\sigma_1(E, T, \omega) = \sigma_d(E, T). \tag{38}$$

Using (12), we get

$$\mathrm{Re}\,\sigma_1(E, T, \omega) = \frac{\mathrm{i}}{2\omega E}\left[j\left(\frac{E}{1+\mathrm{i}\omega}, T\right) - j\left(\frac{E}{1-\mathrm{i}\omega}, T\right)\right]. \tag{39}$$

For numerical computations we present expression (39) in a form

$$\begin{aligned}\mathrm{Re}\,\sigma_1(E, T, \omega) = {}& \frac{1}{1+\omega^2} \\ & - \sqrt{\frac{2}{\pi T}}\frac{[\sinh^2(1/E) + \sin^2(\omega/E)]^{-1}}{\omega E\,\mathrm{erf}(1/\sqrt{2T})}\left[\cosh\frac{1}{E}\sin\frac{\omega}{E}\int_0^1 \exp\left(-\frac{s^2}{2T}\right)\right. \\ & \left.\times\cosh\frac{s}{E}\cos\frac{s\omega}{E}\,\mathrm{d}s - \sinh\frac{1}{E}\cos\frac{\omega}{E}\int_0^1\exp\left(-\frac{s^2}{2T}\right)\sinh\frac{s}{E}\sin\frac{s\omega}{E}\,\mathrm{d}s\right].\end{aligned} \tag{40}$$

At $T \to 0$ from (40) we get the expression presented in [12]:

$$\mathrm{Re}\,\sigma_1(E, 0, \omega) = \frac{1}{1+\omega^2} - \frac{\cosh(1/E)\sin(\omega/E)}{\omega E[\sinh^2(1/E) + \sin^2(\omega/E)]}. \tag{41}$$

The opportunities of creating a terahertz oscillator on Bloch electron oscillations in SLs are defined by conditions of existing negative high-frequency differential conductivity in those regions of current–voltage characteristic where the static differential conductivity is positive [5, 12]. These conditions would prevent development of undesirable domain instabilities (Gunn effect).

Let $\Omega = eEd/\hbar$ be the BO frequency which in normalized measurement units is equal to πE. Then the static differential conductivity σ_d is positive at $\Omega < \Omega_C$ and negative at $\Omega > \Omega_C$, where $\Omega_C = \pi E_C(T) \in (0.914, 1.174)$.

The condition of THz field generation and amplification (see Introduction)

$$\begin{cases}\Omega < \Omega_C \\ \mathrm{Re}\,\sigma_1(\Omega, T, \omega) < 0\end{cases} \tag{42}$$

is very sensitive to the temperature changes as shown in figure 4. The numerical analysis of that condition shows that both inequalities cannot be obeyed simultaneously at $T > 0.01$.

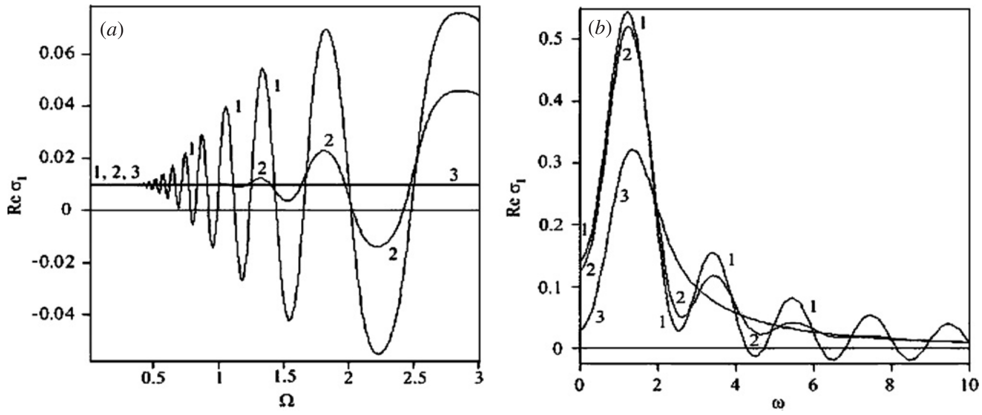

Figure 4. (*a*) Driving field dependence of high-frequency differential conductivity at $\omega = 10$: (1) $T = 0$, $\Omega_C = 1.174$; (2) $T = 0.01$, $\Omega_C = 1.163$; (3) $T = 0.1$, $\Omega_C = 1.06$. (*b*) Dependence of high-frequency differential conductivity on ω at $\Omega = 1$: (1) $T = 0$; (2) $T = 0.01$; (3) $T = 0.1$. At such temperatures the static differential conductivity $\sigma_d = \sigma_1|_{\omega=0}$ is positive.

4. Conclusion

In the present paper, an exact distribution function of the carriers has been found in the lowest parabolic miniband of a SL placed in the dc electric field parallel to the SL axis. The novel formula for the static current density in SL contains temperature dependence, which leads to the current maximum shift to the low field side with increasing temperature.

We have obtained an explicit expression for the high-frequency differential conductivity at arbitrary temperature. It was shown that high-frequency differential conductivity is very sensitive to temperature of SL. We have compared the high-frequency electron behavior at different temperatures and exhibited the drastic change in the character of regions where the high-frequency differential conductivity is negative.

In summary, our analysis shows that SLs with the parabolic miniband dispersion law may be used for the generation and amplification of terahertz fields only at very low temperatures ($T < 0.01\Delta$).

The numerical estimations of the effects predicted are reduced, in general, to measurement units of the electric field and temperature. At $d = 10^{-7}$ cm, $\tau = 10^{-12}$ s, $\Delta \approx 10^{-2}$ eV, we get that units for E and T are $\approx 2 \times 10^3$ V cm^{-1} and $\approx$100 K, respectively. Thus the condition $T < 0.01\Delta$ is equivalent to $T < 1$ K.

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